# Nonlinear Surface Waves on the Interface of Two Non-Kerr-Like Nonlinear Media

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***Abstract*— In recent years, there has been growing interest in studying nonlinear guided-wave propagation as they present potential, yet not fully-explored, applications for high-speed optical signal processing and transmission. In this paper, analytical solutions for nonlinear surface waves on the interface of two nonlinear non-Kerr-like media are derived. The dispersion relations and their relations to the transmission power and initial field distributions are calculated. Several observations are made on the behaviors of the surface waves and their potential applications.**

***Index Terms*—Dispersion, non-Kerr-like media, nonlinear media, surface waves, transmission power.**

## I. Introduction

NONLINEAR guided waves in optical waveguides have recently received growing attention owing to their potential applications to optical signal processing for high-speed communications and optical computing. In the past several years, much research has been concentrated on nonlinear surface waves propagating along the interface between linear and nonlinear planar structures. The reason for selecting the planar structures is that the planar structure is one of the simplest guided-wave structures and is easy for fabrication. A large body of literature has been devoted to dealing with the nonlinear problems (see [1]–[11]).

Among the possible nonlinear modes in a planar structure, self-guided modes are of particular interest to researchers and engineers, mainly because they have now been observed experimentally (see [4]). This leads to a possible way to an all-optical technology in which light can guide and manipulate light itself. For the Kerr-like nonlinear media (where the refractive index of the media is proportional to the square of electric-field intensity), extensive studies have been carried out in the past few years [1], [2].

Although nonlinear optical effects at the boundary between two media have already been investigated for a number of possible geometries with Kerr-like nonlinearity [6]–[9], wave propagation in non-Kerr-like nonlinear media have not been studied in a systematical way. In a practical situation, many materials exhibit a refractive index which varies with the electric-field intensity raised to a power other than two [4]–[11]. That is, a practical medium may not be an exact Kerr-like medium. The actual dependence of the refractive index on the optical field is intimately related to the physical process which gives rise to the nonlinearities. In consequence, practical applications of nonlinear planar structures to an optical device design demand that non-Kerr-like nonlinear structures be studied in details. Some investigations have been done along this line; however, so far, most of them are concentrated on wave propagation in a linear medium bounded by nonlinear claddings or in a nonlinear medium bounded by linear claddings. Little work has been carried out on the wave propagation between two nonlinear media except the work presented by Snyder in [4]. Even in [4], Snyder *et al.* did not solve the nonlinear equation directly or analytically. Rather, they inverted the solution of a linear waveguide to discuss nonlinear surface waves.

To the authors' knowledge, no exact analytical and closed-form solutions for the nonlinear surface waves on the interface of two non-Kerr-like nonlinear media have been reported thus far. In this paper, an attempt is made to directly solve this problem. The analytical solutions are found and the dispersion relations versus different parameters are calculated. Some other interesting results regarding field distributions and transmission power are also obtained.

This paper is organized in the following manner. In Section II, the analytical solutions are derived. In Section III, results and discussions based on the analytical solutions for some cases are presented. Finally, in Section IV, conclusions are drawn.

## II. Field Solutions and Dispersion Relations of the Surface Waves

The structure considered is shown in Fig. 1. It is infinite in both $y$- and $z$-directions. The fields are assumed to be independent of these two coordinates. Two semi-infinite nonlinear media are located in the region $x > 0$ and $x < 0$, respectively. The interface plane is at $x = 0$. The nonlinearities of the two media are represented by the dependence of the permittivities on the field intensities in the following form:

$$\epsilon_i = \epsilon_o(\epsilon_{ri} + \alpha_i \, |\vec{E}|^{\delta_i}), \qquad i = \mathrm{I}, \mathrm{II} \tag{1}$$

where $\epsilon_{ri}$ is the linear part of the dielectric constant of the media and $\delta_i$ represents the nonlinearity, which can be any arbitrary real number. When $\delta_i = 2$, the medium becomes the Kerr-like medium $i = \mathrm{I}$ or II, denoting medium I and II, respectively. $\alpha_i$ is the nonlinear coefficient. If $\alpha$ is positive, the medium becomes a focusing nonlinear medium where the field has its highest intensity at one location and the maximum change of the refractive index is at the same location. If $\alpha$ is negative, it becomes a defocusing nonlinear medium where the field does not have the highest intensity at one location

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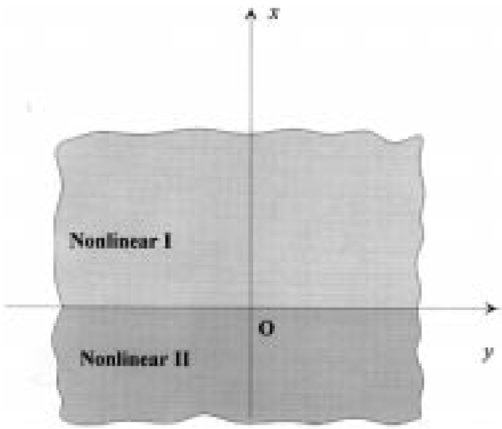


Fig. 1. The discussed structure.

[1]–[11]. The detailed definitions for these two media can be found in [3].

Suppose that the wave is propagating along the direction of the $z$-axis in the form of $e^{j(\omega t-\beta z)}$. Assume that the only nonzero component of electrical fields is the $y$-component

$$E_y(x) = E(x)e^{-j\beta z}. \tag{2}$$

From Maxwell's equations, one can then obtain the magnetic-field components

$$H_x(x) = \frac{1}{j\omega\mu}\frac{\partial E_y}{\partial z}, \qquad H_z = -\frac{1}{j\omega\mu}\frac{\partial E_y}{\partial x}. \tag{3}$$

Note that above equations represent TE modes.

The wave equations for each medium can be written as [3]–[6]

$$\frac{d^2E_i}{dx^2} + [k_o^2(\epsilon_{ri} + \alpha_i|E_i(x)|^{\delta_i}) - \beta^2]E_i(x) = 0. \tag{4}$$

The analytical solution of (4) for each medium, as shown in Fig. 1, can be found in a way similar to that described in [10], [11]. If the medium is a focusing media

$$E_i(x) = \frac{\left[\frac{1}{\alpha_i}\frac{2+\delta_i}{2}\left[\left(\frac{\beta}{k_o}\right)^2 - \epsilon_{ri}\right]\right]^{1/\delta_i}}{\cosh^{2/\delta_i}\left[\pm\frac{\delta_i}{2}\sqrt{\beta^2 - k_o^2\epsilon_{ri}}(x_{0i}-x)\right]}. \tag{5}$$

If the medium is a defocusing medium

$$E_i(x) = \frac{\left[\frac{1}{|\alpha_i|}\frac{2+\delta_i}{2}\left[\left(\frac{\beta}{k_o}\right)^2 - \epsilon_{ri}\right]\right]^{1/\delta_i}}{\sinh^{2/\delta_i}\left[\pm\frac{\delta_i}{2}\sqrt{\beta^2 - k_o^2\epsilon_{ri}}(x_{0i}-x)\right]}. \tag{6}$$

Here, $x_{0i}$ is a constant to be determined by the initial field distributions. It gives the locations of $x$ where the field amplitudes reach maximum in each medium. If the initial field distributions are established in such a way that $x_{0i} = 0$ for both media, the maximum fields will occur at the interface, forming a desired self-focusing surface wave. On the other hand, if a defocusing medium is desired, $x_{0i}$ should not be set to zero. Note here that $x_{0\text{I}}$ and $x_{0\text{II}}$ are not independent from each other as will be shown later.

The above equations can be rewritten, respectively, for each medium:
for medium I ($x > 0$):

$$E_{\text{I}}(x) = A_{\text{I}}/\cosh^{2/\delta_{\text{I}}}\left[\frac{\delta_{\text{I}}}{2}k_{\text{I}}(x_{0\text{I}} - x)\right], \quad \text{when the medium is a focusing medium} \tag{7}$$

$$E_{\text{I}}(x) = A_{\text{I}}/\sinh^{2/\delta_{\text{I}}}\left[\frac{\delta_{\text{I}}}{2}k_{\text{I}}(x_{0\text{I}} - x)\right], \quad \text{when the medium is a defocusing medium} \tag{8}$$

and for medium II ($x < 0$):

$$E_{\text{II}}(x) = A_{\text{II}}/\cosh^{2/\delta_{\text{II}}}\left[\frac{\delta_{\text{II}}}{2}k_{\text{II}}(x - x_{0\text{II}})\right], \quad \text{when the medium is a focusing medium} \tag{9}$$

$$E_{\text{II}}(x) = A_{\text{II}}/\sinh^{2/\delta_{\text{II}}}\left[\frac{\delta_{\text{II}}}{2}k_{\text{II}}(x - x_{0\text{II}})\right], \quad \text{when the medium is a defocusing medium.} \tag{10}$$

Here

$$A_{\text{I}} = \left\{\frac{1}{\alpha_{\text{I}}}\frac{2+\delta_{\text{I}}}{2}\left(\frac{k_{\text{I}}}{k_o}\right)^2\right\}^{1/\delta_{\text{I}}}, \qquad k_{\text{I}}^2 = \beta^2 - k_o^2\epsilon_{r\text{I}} \tag{11}$$

$$A_{\text{II}} = \left\{\frac{1}{\alpha_{\text{II}}}\frac{2+\delta_{\text{II}}}{2}\left(\frac{k_{\text{II}}}{k_o}\right)^2\right\}^{1/\delta_{\text{II}}}, \qquad k_{\text{II}}^2 = \beta^2 - k_o^2\epsilon_{r\text{II}}. \tag{12}$$

Now that the electric fields are obtained as noted above, the magnetic-field components can be found from (3). For instance, for $H_z$, one can have the following:
for medium I:

$$H_{z\text{I}}(x) = -\frac{1}{j\omega\mu}\frac{dE_{\text{I}}(x)}{dx} = -\frac{1}{j\omega\mu}k_{\text{I}}\tanh\left[\frac{\delta_{\text{I}}}{2}k_{\text{I}}(x_{0\text{I}} - x)\right]E_{\text{I}}(x), \quad \text{when the medium is a focusing medium} \tag{13}$$

$$H_{z\text{I}}(x) = -\frac{1}{j\omega\mu}\frac{dE_{\text{I}}(x)}{dx} = -\frac{1}{j\omega\mu}k_{\text{I}}\,\text{ctanh}\left[\frac{\delta_{\text{I}}}{2}k_{\text{I}}(x_{0\text{I}} - x)\right]E_{\text{I}}(x), \quad \text{when the medium is a defocusing medium} \tag{14}$$

and for medium II:

$$H_{z\text{II}}(x) = -\frac{1}{j\omega\mu}\frac{dE_{\text{II}}(x)}{dx} = \frac{1}{j\omega\mu}k_{\text{II}}\tanh\left[\frac{\delta_{\text{II}}}{2}k_{\text{II}}(x - x_{0\text{II}})\right]E_{\text{II}}(x), \quad \text{when the medium is a focusing medium} \tag{15}$$

$$H_{z\text{II}}(x) = -\frac{1}{j\omega\mu}\frac{dE_{\text{II}}(x)}{dx} = \frac{1}{j\omega\mu}k_{\text{II}}\,\text{ctanh}\left[\frac{\delta_{\text{II}}}{2}k_{\text{II}}(x - x_{0\text{II}})\right]E_{\text{II}}(x), \quad \text{when the medium is a defocusing medium.} \tag{16}$$

At the interface $x=0$, boundary conditions require that $E_y$ and $H_z \sim \frac{dE_y}{dx}|_{x=0}$ be continuous across the interface. As a result, the following conditions must be satisfied:

$$E_{y\mathrm{I}}(x=0)=E_{y\mathrm{II}}(x=0) \tag{17}$$

$$H_{z\mathrm{I}}(x=0)=H_{z\mathrm{II}}(x=0). \tag{18}$$

Application of the above boundary conditions leads to the following dispersion equations for four combinations of different media:

*Case I:* when both media are focusing media

$$k_\mathrm{I}\tanh\left[\frac{\delta_\mathrm{I}}{2}k_\mathrm{I}x_{0\mathrm{I}}\right] = k_\mathrm{II}\tanh\left[\frac{\delta_\mathrm{II}}{2}k_\mathrm{II}x_{0\mathrm{II}}\right] \tag{19}$$

$$\frac{A_\mathrm{I}}{\left(\cosh\left[\frac{\delta_\mathrm{I}}{2}\sqrt{\beta^2-k_o^2\epsilon_{r\mathrm{I}}}x_{0\mathrm{I}}\right]\right)^{2/\delta_\mathrm{I}}} = \frac{A_\mathrm{II}}{\left(\cosh\left[\frac{\delta_\mathrm{II}}{2}\sqrt{\beta^2-k_o^2\epsilon_{r\mathrm{II}}}x_{0\mathrm{II}}\right]\right)^{2/\delta_\mathrm{II}}}. \tag{20}$$

*Case II:* when both media are defocusing media

$$k_\mathrm{I}\,\mathrm{ctanh}\left[\frac{\delta_\mathrm{I}}{2}k_\mathrm{I}x_{0\mathrm{I}}\right] = k_\mathrm{II}\mathrm{ctanh}\left[\frac{\delta_\mathrm{II}}{2}k_\mathrm{II}x_{0\mathrm{II}}\right] \tag{21}$$

$$\frac{A_\mathrm{I}}{\left(\sinh\left[\frac{\delta_\mathrm{I}}{2}\sqrt{\beta^2-k_o^2\epsilon_{r\mathrm{I}}}x_{0\mathrm{I}}\right]\right)^{2/\delta_\mathrm{I}}} = \frac{A_\mathrm{II}}{\left(\sinh\left[\frac{\delta_\mathrm{II}}{2}\sqrt{\beta^2-k_o^2\epsilon_{r\mathrm{II}}}x_{0\mathrm{II}}\right]\right)^{2/\delta_\mathrm{II}}}. \tag{22}$$

*Case III:* when medium I is defocusing while medium II is focusing

$$k_\mathrm{I}\,\mathrm{ctanh}\left[\frac{\delta_\mathrm{I}}{2}k_\mathrm{I}x_{0\mathrm{I}}\right] = k_\mathrm{II}\tanh\left[\frac{\delta_\mathrm{II}}{2}k_\mathrm{II}x_{0\mathrm{II}}\right] \tag{23}$$

$$\frac{A_\mathrm{I}}{\left(\sinh\left[\frac{\delta_\mathrm{I}}{2}\sqrt{\beta^2-k_o^2\epsilon_{r\mathrm{I}}}x_{0\mathrm{I}}\right]\right)^{2/\delta_\mathrm{I}}} = \frac{A_\mathrm{II}}{\left(\cosh\left[\frac{\delta_\mathrm{II}}{2}\sqrt{\beta^2-k_o^2\epsilon_{r\mathrm{II}}}x_{0\mathrm{II}}\right]\right)^{2/\delta_\mathrm{II}}}. \tag{24}$$

*Case IV:* when medium I is focusing while medium II is defocusing

$$k_\mathrm{I}\tanh\left[\frac{\delta_\mathrm{I}}{2}k_\mathrm{I}x_{0\mathrm{I}}\right] = k_\mathrm{II}\mathrm{ctanh}\left[\frac{\delta_\mathrm{II}}{2}k_\mathrm{II}x_{0\mathrm{II}}\right] \tag{25}$$

$$\frac{A_\mathrm{I}}{\left(\cosh\left[\frac{\delta_\mathrm{I}}{2}\sqrt{\beta^2-k_o^2\epsilon_{r\mathrm{I}}}x_{0\mathrm{I}}\right]\right)^{2/\delta_\mathrm{I}}} = \frac{A_\mathrm{II}}{\left(\sinh\left[\frac{\delta_\mathrm{II}}{2}\sqrt{\beta^2-k_o^2\epsilon_{r\mathrm{II}}}x_{0\mathrm{II}}\right]\right)^{2/\delta_\mathrm{II}}}. \tag{26}$$

The above dispersion relations for focusing, defocusing, or mixed focusing and defocusing media can, respectively, be rewritten in a simplified form as follows:

*Case I:*

$$\left\{\frac{\frac{2+\delta_\mathrm{I}}{2}\frac{N}{\alpha_\mathrm{I}}}{\cosh^2\left[\frac{\delta_\mathrm{I}}{2}k_o\sqrt{N}x_{0\mathrm{I}}\right]}\right\}^{1/\delta_\mathrm{I}} = \left\{\frac{\frac{2+\delta_\mathrm{II}}{2}\frac{N+\Delta}{\alpha_\mathrm{II}}}{\cosh^2\left[\frac{\delta_\mathrm{II}}{2}k_o\sqrt{N+\Delta}x_{0\mathrm{II}}\right]}\right\}^{1/\delta_\mathrm{II}} \tag{27}$$

$$\sqrt{N}\tanh\left[\frac{\delta_\mathrm{I}}{2}k_o\sqrt{N}x_{0\mathrm{I}}\right] = \sqrt{N+\Delta}\tanh\left[\frac{\delta_\mathrm{II}}{2}k_o\sqrt{N+\Delta}x_{0\mathrm{II}}\right]. \tag{28}$$

*Case II:*

$$\left\{\frac{\frac{2+\delta_\mathrm{I}}{2}\frac{N}{\alpha_\mathrm{I}}}{\sinh^2\left[\frac{\delta_\mathrm{I}}{2}k_o\sqrt{N}x_{0\mathrm{I}}\right]}\right\}^{1/\delta_\mathrm{I}} = \left\{\frac{\frac{2+\delta_\mathrm{II}}{2}\frac{N+\Delta}{\alpha_\mathrm{II}}}{\sinh^2\left[\frac{\delta_\mathrm{II}}{2}k_o\sqrt{N+\Delta}x_{0\mathrm{II}}\right]}\right\}^{1/\delta_\mathrm{II}} \tag{29}$$

$$\sqrt{N}\,\mathrm{ctanh}\left[\frac{\delta_\mathrm{I}}{2}k_o\sqrt{N}x_{0\mathrm{I}}\right] = \sqrt{N+\Delta}\,\mathrm{ctanh}\left[\frac{\delta_\mathrm{II}}{2}k_o\sqrt{N+\Delta}x_{0\mathrm{II}}\right]. \tag{30}$$

*Case III:*

$$\left\{\frac{\frac{2+\delta_\mathrm{I}}{2}\frac{N}{\alpha_\mathrm{I}}}{\sinh^2\left[\frac{\delta_\mathrm{I}}{2}k_o\sqrt{N}x_{0\mathrm{I}}\right]}\right\}^{1/\delta_\mathrm{I}} = \left\{\frac{\frac{2+\delta_\mathrm{II}}{2}\frac{N+\Delta}{\alpha_\mathrm{II}}}{\cosh^2\left[\frac{\delta_\mathrm{II}}{2}k_o\sqrt{N+\Delta}x_{0\mathrm{II}}\right]}\right\}^{1/\delta_\mathrm{II}} \tag{31}$$

$$\sqrt{N}\,\mathrm{ctanh}\left[\frac{\delta_\mathrm{I}}{2}k_o\sqrt{N}x_{0\mathrm{I}}\right] = \sqrt{N+\Delta}\tanh\left[\frac{\delta_\mathrm{II}}{2}k_o\sqrt{N+\Delta}x_{0\mathrm{II}}\right]. \tag{32}$$

*Case IV:*

$$\left\{\frac{\frac{2+\delta_\mathrm{I}}{2}\frac{N}{\alpha_\mathrm{I}}}{\cosh^2\left[\frac{\delta_\mathrm{I}}{2}k_o\sqrt{N}x_{0\mathrm{I}}\right]}\right\}^{1/\delta_\mathrm{I}} = \left\{\frac{\frac{2+\delta_\mathrm{II}}{2}\frac{N+\Delta}{\alpha_\mathrm{II}}}{\sinh^2\left[\frac{\delta_\mathrm{II}}{2}k_o\sqrt{N+\Delta}x_{0\mathrm{II}}\right]}\right\}^{1/\delta_\mathrm{II}} \tag{33}$$

$$\sqrt{N}\tanh\left[\frac{\delta_\mathrm{I}}{2}k_o\sqrt{N}x_{0\mathrm{I}}\right] = \sqrt{N+\Delta}\,\mathrm{ctanh}\left[\frac{\delta_\mathrm{II}}{2}k_o\sqrt{N+\Delta}x_{0\mathrm{II}}\right] \tag{34}$$

where

$$\alpha=\frac{\alpha_\mathrm{II}}{\alpha_\mathrm{I}}$$
$$N=\left(\frac{\beta}{k_o}\right)^2-\epsilon_{r\mathrm{I}}$$
$$\Delta=\epsilon_{r\mathrm{I}}-\epsilon_{r\mathrm{II}}\ge 0. \tag{35}$$

Now the feasibility of Case IV will be examined. Rewriting (34), one has

$$\frac{\operatorname{ctanh}\left[\frac{\delta_{\mathrm{II}}}{2}k_o\sqrt{N+\Delta}x_{0\mathrm{II}}\right]}{\tanh\left[\frac{\delta_{\mathrm{I}}}{2}k_o\sqrt{N}x_{0\mathrm{I}}\right]}=\sqrt{\frac{N}{N+\Delta}}<1. \tag{36}$$

Since the left-side of (36) is always larger than one, (34) has no rational solutions. Therefore, there exist no surface waves in Case IV. Note that the assumption of $\Delta \geq 0$ is without the loss of generality.

For the remaining three cases, the dispersion relations of the surface waves are then determined by (27)–(32). From these equations, one can see that given the operating frequency and the material parameters $x_{01}$, $x_{02}$, and $\beta$ are not independent of each other. In each case, there are two different dispersion equations. Therefore, only one of $x_{01}$, $x_{02}$, and $\beta$ is free and to be determined by other conditions, such as initial conditions. Once it is set, the other two quantities are determined.

For Case I, further simplification of (28) reads

$$\frac{\tanh\left[\frac{\delta_{\mathrm{II}}}{2}k_o\sqrt{N+\Delta}x_{0\mathrm{II}}\right]}{\tanh\left[\frac{\delta_{\mathrm{I}}}{2}k_o\sqrt{N}x_{0\mathrm{I}}\right]}=\sqrt{\frac{N}{N+\Delta}}\leq 1 \tag{37}$$

which leads to

$$\frac{\delta_{\mathrm{II}}}{2}k_o\sqrt{N+\Delta}x_{0\mathrm{II}}\leq\frac{\delta_{\mathrm{I}}}{2}k_o\sqrt{N}x_{0\mathrm{I}},$$

or

$$\frac{\delta_{\mathrm{II}}x_{0\mathrm{II}}}{\delta_{\mathrm{I}}x_{0\mathrm{I}}}\leq\sqrt{\frac{N}{N+\Delta}}\leq 1 \tag{38}$$

or

$$\delta_{\mathrm{I}}x_{0\mathrm{I}}\geq\delta_{\mathrm{II}}x_{0\mathrm{II}}. \tag{39}$$

That is, in Case I (where both media are focusing media), for a surface wave to propagate, initial conditions $x_{0i}$ and medium parameters $\delta_i$ must satisfy (39).

## III. Numerical Results and Discussions

To obtain the insight into the propagation characteristics associated with the surface waves, numerical calculations have been performed using the analytical solutions solved above. This brings about interesting phenomena which is new and important for an actual design. The general dispersions and field distributions are quite complicated. Therefore, in the following, only special cases are considered for simplicity.

### A. $x_{0I}=0$ and $\delta_I=\delta_{II}$

From (14) and (16), it can be seen that for a defocusing medium, the constant $x_{0i}$ can not be zero (otherwise, $H_z$ will have infinite values at $x=0$). That is, the maximum electric-field points could not be on the interface of the two defocusing media. However, for focusing media, $x_{0\mathrm{I}}=0$ and $x_{0\mathrm{II}}=0$ are possible. Take Case I as an example. If $x_{0\mathrm{I}}=x_{0\mathrm{II}}=0$, the dispersion relations, (27) and (28), reduce to

$$\left\{\frac{2+\delta_{\mathrm{I}}}{2}\frac{N}{\alpha_{\mathrm{I}}}\right\}^{1/\delta_{\mathrm{I}}}=\left\{\frac{2+\delta_{\mathrm{II}}}{2}\frac{N+\Delta}{\alpha_{\mathrm{II}}}\right\}^{1/\delta_{\mathrm{II}}}. \tag{40}$$

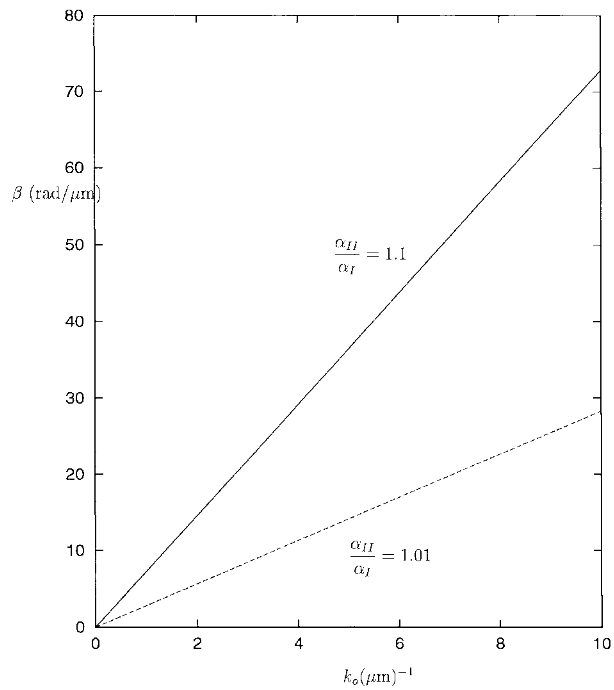


Fig. 2. The dispersion curves for different $\alpha_{\mathrm{II}}/\alpha_{\mathrm{I}}$ under the conditions $\epsilon_{\mathrm{I}}=3$, $\Delta=0.5$, $x_{0\mathrm{I}}=x_{0\mathrm{II}}=0$, and $\delta_{\mathrm{I}}=\delta_{\mathrm{II}}$.

Furthermore, if $\delta_{\mathrm{I}}=\delta_{\mathrm{II}}$, it becomes

$$N=\frac{\Delta}{\frac{\alpha_{\mathrm{II}}}{\alpha_{\mathrm{I}}}-1}. \tag{41}$$

The dispersion relation becomes independent of $\delta_{\mathrm{I}}(\delta_{\mathrm{II}})$.

Suppose that $\alpha_{\mathrm{I}}<\alpha_{\mathrm{II}}$. Then

$$\beta=k_o\sqrt{\epsilon_{r\mathrm{I}}+\frac{\Delta}{\frac{\alpha_{\mathrm{II}}}{\alpha_{\mathrm{I}}}-1}}. \tag{42}$$

The dispersion relation becomes linear with frequency $k_o$. Its slope, the phase velocity of the surface wave, becomes a constant, which is only dependent on material parameters $\epsilon_{r\mathrm{I}}, \epsilon_{r\mathrm{II}}, \alpha_{\mathrm{II}}/\alpha_{\mathrm{I}}$. As a result, signals propagating through the interface of the two nonlinear media will not suffer any distortion and the media become distortionless media.

Fig. 2 shows the propagation constant $\beta$ versus frequency with various $\alpha_{\mathrm{II}}/\alpha_{\mathrm{I}}$ in the case. Note that the propagation is independent of the value of $\delta_{\mathrm{I}}$ and $\delta_{\mathrm{II}}$ as long as they are equal. Figs. 3 and 4 show the related field distributions. As seen, the fields concentrate mainly in the neighborhood of the interface, forming the surface waves. In addition, as $\alpha_{\mathrm{II}}/\alpha_{\mathrm{I}}$ becomes smaller or $\delta$ becomes larger, the field concentration is intensified. Consequently, nonlinear media practically need not be semi-infinite. The region which contains little field energy can be removed without affecting the propagation properties.

Fig. 5 shows the propagation constant $\beta$ versus $\alpha_{\mathrm{II}}/\alpha_{\mathrm{I}}$ for various $k_o$. It indicates that for a given $k_o$, if $\alpha_{\mathrm{II}}/\alpha_{\mathrm{I}}$ is large enough, $\beta$ becomes almost independent of $\alpha_{\mathrm{II}}/\alpha_{\mathrm{I}}$.

### B. $\Delta=0$ ($\epsilon_{rI}=\epsilon_{rII}$)

If $\Delta=0(\epsilon_{r\mathrm{I}}=\epsilon_{r\mathrm{II}})$, (32) has no real solutions. For Case III, where medium I is defocusing and medium II is focusing, the

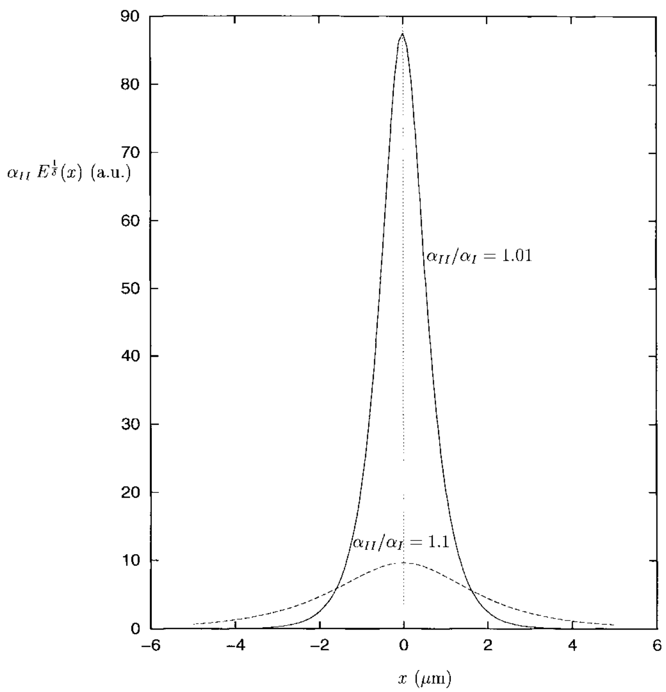


Fig. 3. The electric-field distribution for $\delta_{\rm I} = \delta_{\rm II} = \delta = 1.5$ and various $\alpha_{\rm II}/\alpha_{\rm I}$. Here $\epsilon_{\rm I} = 3$, $\Delta = 0.5$, $x_{0\rm I} = x_{0\rm II} = 0$, $k_o = 0.4\ \mu{\rm m}^{-1}$.

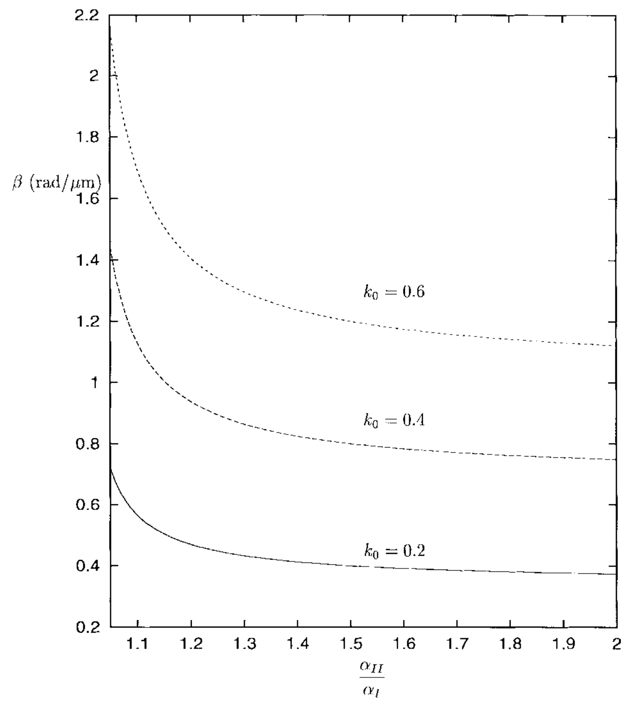


Fig. 5. The propagation constant $\beta$ versus $\alpha_{\rm II}/\alpha_{\rm I}$ for various $k_o$. Here $\epsilon_{\rm I} = 3$, $\Delta = 0.5$, $x_{0\rm I} = x_{0\rm II} = 0$, and $\delta_{\rm I} = \delta_{\rm II}$.

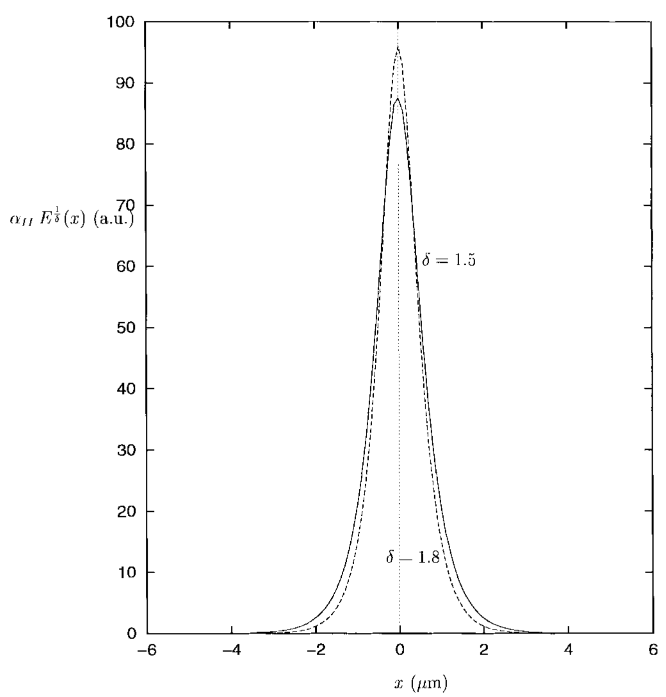


Fig. 4. The electric-field distribution for $\delta_{\rm I} = \delta_{\rm II} = \delta$. Here $\epsilon_{\rm I} = 3$, $\Delta = 0.5$, $x_{0\rm I} = x_{0\rm II} = 0$, and $\alpha_{\rm II}/\alpha_{\rm I} = 1.01$.

surface waves do not exist. However, (28) and (30) have real solutions (i.e., surface waves can exist for Cases I and II). In fact, when $\delta_{\rm I}x_{0\rm I} = \delta_{\rm II}x_{0\rm II}$, the two equation are automatically satisfied. The dispersion relations are then solely determined by (27) and (29), with $\delta_{\rm I}x_{0\rm I} = \delta_{\rm II}x_{0\rm II}$. In the following, Case I is computed.

For Case I (both media are focusing), one has

$$\left\{\frac{\frac{2+\delta_{\rm I}}{2}\frac{N}{\alpha_{\rm I}}}{\cosh^2\left(\frac{\delta_{\rm I}}{2}k_o\sqrt{N}x_{0\rm I}\right)}\right\}^{\frac{1}{\delta_{\rm I}}} = \left\{\frac{\frac{2+\delta_{\rm II}}{2}\frac{N}{\alpha_{\rm II}}}{\cosh^2\left(\frac{\delta_{\rm I}}{2}k_o\sqrt{N}x_{0\rm I}\right)}\right\}^{\frac{1}{\delta_{\rm II}}}. \tag{43}$$

Rewriting it gives

$$\frac{\left[\frac{2+\delta_{\rm I}}{2\alpha_{\rm I}}\right]^{\frac{1}{\delta_{\rm I}}}}{\left[\frac{2+\delta_{\rm II}}{2\alpha_{\rm II}}\right]^{\frac{1}{\delta_{\rm II}}}}N^{\frac{1}{\delta_{\rm I}}-\frac{1}{\delta_{\rm II}}} = \left[\cosh\left(\frac{\delta_{\rm I}}{2}k_o\sqrt{N}x_{0\rm I}\right)\right]^{\frac{2}{\delta_{\rm I}}-\frac{2}{\delta_{\rm II}}}$$

or

$$k_o = \frac{2}{\delta_{\rm I}}\frac{1}{\sqrt{N}x_{0\rm I}}{\rm arcosh} \cdot\left(\sqrt{N\left[\frac{2\alpha_{\rm II}}{2+\delta_{\rm II}}\right]^{\frac{\delta_{\rm I}}{\delta_{\rm II}-\delta_{\rm I}}}\left[\frac{2+\delta_{\rm I}}{2\alpha_{\rm I}}\right]^{\frac{\delta_{\rm II}}{\delta_{\rm II}-\delta_{\rm I}}}}\right). \tag{44}$$

Fig. 6 shows the dispersion curves for $\epsilon_{r\rm I} = \epsilon_{r\rm II}$, $x_{0\rm I} = 0.8$, $\delta_{\rm I} = 2$, $\delta_{\rm II} = \delta\delta_{\rm I}$. It is seen that for each $k_o$, there exists two different values of $\beta$. Thus, two possible modes can propagate with the same frequency but different phase velocity. There also exists a maximum frequency beyond which surface waves cannot propagate in the structure.

### *C. Transmission Power for Case I (Both Media Being Focusing Media)*

From the field solution (5), one can compute the transmission power for Case I as follows:

$$\alpha_{\rm II}^{2/\delta_{\rm II}}P = \left(\frac{\alpha_{\rm I}^{1/\delta_{\rm I}}}{\alpha_{\rm II}^{1/\delta_{\rm II}}}\right)^2\int_0^\infty\left\{\left[\frac{\frac{2+\delta_{\rm I}}{2}N}{\cosh^2\left[\frac{\delta_{\rm I}}{2}k_o\sqrt{N}(x_{0\rm I}-x)\right]}\right]^{\frac{2}{\delta_{\rm I}}}\right.$$

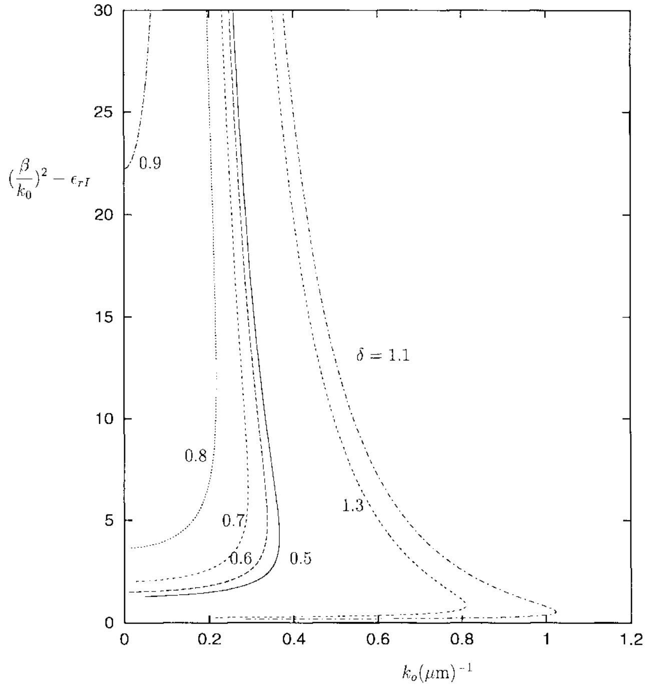


Fig. 6. Dispersion curves for $\epsilon_{\rm I} = \epsilon_{\rm II}$. Here $x_{0\rm I} = 0.8$, $\alpha_{\rm II}^{1/\delta_{\rm II}}/\alpha_{\rm I}^{1/\delta_{\rm I}} = 1.2$, $\delta = \delta_{\rm II}/\delta_{\rm I}$, $\delta_{\rm I} = 2$.

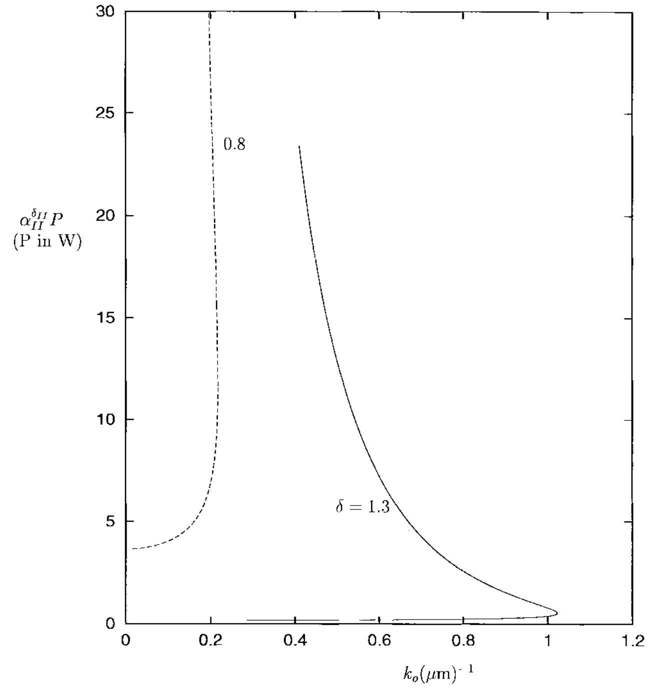


Fig. 7. Transmission power versus $k_o$ for $\epsilon_{\rm I} = \epsilon_{\rm II}$. Here $x_{0\rm I} = 0.8$, $\alpha_{\rm II}^{1/\delta_{\rm II}}/\alpha_{\rm I}^{1/\delta_{\rm I}} = 1.2$, $\delta = \delta_{\rm II}/\delta_{\rm I}, \delta_{\rm I} = 2$.

$$+ \left[\frac{\frac{2+\delta_{\rm II}}{2} N}{\cosh^2\left[\frac{\delta_{\rm II}}{2} k_o \sqrt{N}(x_{0\rm II} + x)\right]}\right]^{\frac{2}{\delta_{\rm II}}} \Bigg\} dx. \quad (45)$$

Fig. 7 shows the transmission power $\alpha_{\rm II}^{2/\delta_{\rm II}} P$ versus $k_o$. Unlike in the linear media, the transmission power is now related strongly to the frequency. For a given exciting frequency, there are two possible transmission powers corresponding to the two modes as indicated before.

By comparing Fig. 6 with Fig. 7, one can find that frequency, transmission power, and propagation constant in the nonlinear structures are related to each other. If one of them is given, the other two are determined. For example, if the frequency is given, the propagation constant and transmission power can be found (from the two figures). If the propagation constant is given, the frequency and transmission power can be found (from the figures). If the transmission power is given, the propagation constant and frequency can then be found. In other words, Fig. 6 along with Fig. 7 gives one the look-up figures for designing the nonlinear transmission media.

## IV. Conclusion

In this paper, surface waves in the structure consisting of two power-law nonlinear media are studied. Analytical expressions for the surface waves are obtained and some cases are calculated. If the initial conditions are developed in such a way that the maximum field points are at the interface of the two nonlinear media (i.e., $x_{0\rm I} = x_{0\rm II} = 0$), the media become distortionless. In this case, any mode with arbitrary frequency can be transmitted by the structure without distortions and there is no cutoff frequency. The field distributions are dependent only on the material parameters. For the other cases, the situations are different. For a given frequency there may exist two related propagation constants $\beta$, or modes. They propagate with different phase velocities. However, not all of the frequencies can form the modes and propagate in the structure. There is a critical frequency for given material parameters. Below it, two modes can exist in the structure; above it, they cannot propagate. In addition, transmission power, propagation constant, and frequency are dependent upon each other. These results are useful for designing a possible new optical devices based on the nonlinear waveguide structure.

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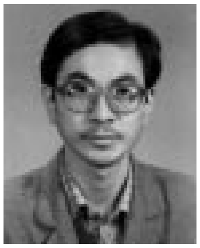

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